# Fluid Mechanics

Formula Collection

Anyone measuring gas flow works with the same relationships day after day. This collection brings them together – from the law of friction to the pipe friction factor.

## The nine chapters

- **Fluid properties** – density, compressibility, gas equation
- **Viscosity** – law of friction, speed of sound
- **Fluid statics** – pressure distribution, manometer, buoyancy
- **Conservation laws** – continuity and Bernoulli
- **Consequences** – Pitot tube, diffuser, discharge
- **Momentum** – forces on components carrying flow
- **Pipe flow** – Reynolds, laminar, turbulent
- **Pressure losses** – loss coefficients of fittings
- **Units** – conversions and constants

## How to use it

Every numbered equation sits in a grey box and carries its number on the right. Below it, a table lists each symbol with meaning and unit.

The blue boxes name the instrument that rests on each relationship, and what matters when applying it.

All calculations use SI units. Conversion to bar, mbar and $m^3/h$ is given in chapter 9.

WHY WE PUBLISH THIS

### A measurement is only as good as the model behind it.

We calibrate, measure and build test systems for air and gases. This physics is the language we share with our customers.

## Symbols and conventions

| | |
|---|---|
| **Unit system** | SI throughout. Pressures are **absolute pressures** unless stated otherwise. |
| **Velocity** | $c$ is the local velocity, $\bar{c}$ the mean over the cross section. Where the distinction does not matter, $c$ is used. |
| **Subscripts 1 and 2** | State upstream and downstream of the point under consideration, counted in flow direction. |

## List of symbols

| | | | |
|---|---|---|---|
| $a$ | speed of sound, acceleration | $s$ | gap width, wall thickness, path |
| $A$ | area, cross section | $T$ | absolute temperature |
| $c$ | velocity | $U$ | wetted perimeter |
| $\bar{c}$ | mean velocity | $V, \dot{V}$ | volume, volume flow rate |
| $D$ | pipe diameter, shear rate | $Y_V$ | specific loss energy |
| $d_{hyd}$ | hydraulic diameter | $z$ | height coordinate |
| $E$ | bulk modulus | $\alpha$ | contraction coefficient, angle of inclination |
| $F$ | force | $\delta$ | angle of impact |
| $g$ | gravitational acceleration | $\eta$ | dynamic viscosity |
| $H_V$ | head loss | $\eta_D$ | diffuser efficiency |
| $k_s$ | sand roughness | $\vartheta$ | temperature in °C, opening angle |
| $K$ | compressibility | $\kappa$ | isentropic exponent |
| $L$ | length | $\lambda$ | pipe friction factor |
| $m, \dot{m}$ | mass, mass flow rate | $\nu$ | kinematic viscosity |
| $n$ | rotational speed | $\rho$ | density |
| $p$ | pressure | $\sigma$ | stress in the wall |
| Re | Reynolds number | $\tau$ | shear stress |
| $R, r$ | radius | $\upsilon$ | specific volume |
| $R_i$ | specific gas constant | $\zeta$ | loss coefficient |
| | | $\omega$ | angular velocity |

Symbols carrying more than one meaning are common in fluid mechanics – *D* stands for pipe diameter and for shear rate, *a* for the speed of sound and for an acceleration. Which one applies is stated in the symbol table below each equation.

# 1 Properties of the fluid

Four quantities cover most tasks: density, compressibility, viscosity and temperature. In gases the first two depend so strongly on the state that they have to be calculated.

## 1.1 Compressibility

Compressibility is the relative change in volume per change in pressure. Liquids and gases differ in it by orders of magnitude.

$$\frac{\Delta p}{E_{Fl}} = -\frac{\Delta V}{V_0} \qquad \text{with} \qquad K = \frac{1}{E_{Fl}} \tag{1.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\Delta p$ | change in pressure | Pa |
| $\Delta V$ | change in volume | $m^3$ |
| $V_0$ | initial volume | $m^3$ |
| $E_{Fl}$ | bulk modulus of the liquid | Pa |
| $K$ | compressibility | $Pa^{-1}$ |

For a gas the initial pressure itself takes the place of the bulk modulus as long as the temperature stays constant: isothermally, a gas is as stiff as the pressure it is held at. Under fast, isentropic compression, as in a sound wave, it is $\kappa \cdot p_0$ (section 2.3).

$$\frac{\Delta p}{p_0} = -\frac{\Delta V}{V_0} \qquad \text{hence} \qquad E_{Gas} = p_0 \tag{1.2}$$

In the standard state $E_{Gas}$ = 1.01325 bar. The minus sign records that rising pressure reduces the volume.

## 1.2 Ideal gas equation

It links pressure, volume and temperature. Every conversion of an actual volume flow rate to reference conditions runs through it.

$$p \cdot V = m \cdot R_i \cdot T \tag{1.3}$$

$$p \cdot v = R_i \cdot T \qquad \text{with} \qquad v = \frac{1}{\rho}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p$ | absolute pressure | Pa |
| $V$ | volume | $m^3$ |
| $m$ | mass of the gas | kg |
| $R_i$ | specific gas constant; dry air 287.05 | $J\,kg^{-1}\,K^{-1}$ |
| $T$ | absolute temperature | K |
| $\rho$ | density | $kg\,m^{-3}$ |
| $v$ | specific volume | $m^3\,kg^{-1}$ |

**WHY THE REFERENCE CONDITION IS PART OF THE FIGURE**

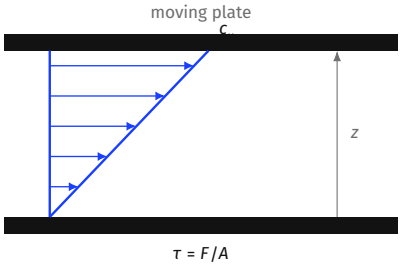
The same gas fills roughly 7 % more space at 20 °C than at 0 °C. A volume flow rate therefore needs pressure and temperature with it. Our data sheets carry the reference condition as a line of its own – for the capillary calibrated leaks of the KL series either 1000 mbar, 20 °C and 0 % r. h. or 1013.25 mbar, 0 °C and 0 % r. h. Which condition applies is stated in the data sheet of the individual instrument.

The second is the *standard state* of physics – 0 °C and 1013.25 mbar. Confusing the two costs almost 9 %.

## 2 Viscosity and speed of sound

Viscosity is a fluid's internal resistance to deformation. It separates laminar from turbulent flow and carries the laminar flow element.

### 2.1 Dynamic and kinematic viscosity




Shear flow between a stationary and a moving plate. The velocity profile is linear and the shear stress constant across the gap.

$$\eta = \frac{\tau}{\mathrm{d}c_x/\mathrm{d}z} \qquad \nu = \frac{\eta}{\rho} \tag{2.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\eta$ | dynamic viscosity | Pa s |
| $\nu$ | kinematic viscosity | $m^2\,s^{-1}$ |
| $\tau$ | shear stress, $\tau = F/A$ | Pa |
| $F$ | friction force | N |
| $A$ | contact area of the layers | $m^2$ |
| $\mathrm{d}c_x/\mathrm{d}z$ | shear rate across the flow | $s^{-1}$ |

Kinematic viscosity is the dynamic one referred to density. In this form it appears in the Reynolds number.

### 2.2 Newtonian law of friction

A fluid is called Newtonian when shear stress is proportional to shear rate. Air and the technical gases are, to a good approximation.

$$\tau = \eta \cdot D = \eta \cdot \frac{\Delta c_x}{\Delta z} \tag{2.2}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $D$ | shear rate | $s^{-1}$ |

### 2.3 Speed of sound

It is the speed at which a pressure disturbance travels through the fluid, and the limit at which a nozzle chokes. In liquids it follows from the bulk modulus, in gases from temperature and isentropic exponent. Pressure does not enter.

$$a = \sqrt{\frac{E_{Fl}}{\rho_0}} \qquad a = \sqrt{\kappa \cdot R_i \cdot T} \tag{2.3}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $a$ | speed of sound | $m\,s^{-1}$ |
| $\kappa$ | isentropic exponent; air ≈ 1.4 | – |
| $\rho_0$ | density in the initial state | $kg\,m^{-3}$ |

**WHERE THE SPEED OF SOUND LIMITS THE MEASUREMENT**

Once a gas reaches the speed of sound in the narrowest cross section, lowering the downstream pressure further does not increase the mass flow rate. The critical flow nozzle uses this effect as a standard; for every other differential pressure device it sets the upper limit of the operating range. In air at 20 °C the speed of sound is 343 m/s.

## 3 Fluid statics

In a fluid at rest there is no shear stress. What remains is pressure, and that depends on height alone. Pressure measurement with a liquid column rests on this relationship.

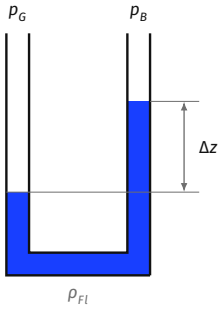
### 3.1 Pressure distribution in liquids

$$p(z) = p_0 - \rho \cdot g \cdot z \qquad p(z) = p_B + \rho \cdot g \cdot |z| \tag{3.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p_0$ | pressure at the origin of the $z$ axis | Pa |
| $p_B$ | barometric pressure at the surface | Pa |
| $z$ | height coordinate | m |
| $g$ | gravitational acceleration, 9.81 | $m\,s^{-2}$ |

The left form counts $z$ upward from the bottom, the right one downward from the surface. The form that matches the chosen coordinate system applies.

### 3.2 U-tube manometer




The height difference of the liquid column is the measurement signal. It depends only on the density of the sealing liquid, not on the bore of the legs.

$$p_G = p_B + \rho_{Fl} \cdot g \cdot \Delta z \tag{3.2}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p_G$ | pressure of the gas to be measured | Pa |
| $\rho_{Fl}$ | density of the sealing liquid | kg m$^{-3}$ |
| $\Delta z$ | height difference of the two columns | m |

**FROM THE U-TUBE TO THE DIFFERENTIAL PRESSURE SENSOR**

Every differential pressure measurement measures what this U-tube measures, with a sensor in place of a liquid column. The differential pressure sensors in our systems are calibrated traceably; the chain ends at a pressure standard and thus at a height and a density.

### 3.3 Buoyancy

$$F_a = \rho \cdot g \cdot V \tag{3.3}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $F_a$ | buoyant force | N |
| $\rho$ | density of the surrounding fluid | kg m$^{-3}$ |
| $V$ | displaced volume | m$^3$ |

$V$ is the submerged part of the body. When weighing in air, buoyancy reduces the indicated mass by 1.2 g per litre of volume.

### 3.4 Barometric height formula

It gives air pressure against height at constant temperature. Up to 1000 m the approximation holds; above that, the stratification of the atmosphere has to be taken into account.

$$p(z) = p_{b0} \cdot e^{-\left(\frac{\rho_{b0}}{p_{b0}} \cdot g \cdot z\right)} \tag{3.4}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p(z)$ | air pressure at height $z$ | Pa |
| $p_{b0}$ | pressure at sea level, 101 325 | Pa |
| $\rho_{b0}$ | air density at sea level, 1.225 | kg m$^{-3}$ |
| $T_{b0}$ | temperature at sea level, 288.15 | K |

The three values are the ground-level conditions of the ICAO standard atmosphere. The gas equation links them: $\rho_{b0}/p_{b0} = 1/(R_i \cdot T_{b0})$.

### 3.5 Pressure force and accelerated vessels

The force acting on a curved surface in one coordinate direction equals the force on its projection in that direction. Barlow's formula follows.

$$F_z = p \cdot A_{proj} \qquad F_W = \sigma_{zul} \cdot s \cdot L \tag{3.5}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $F_z$ | pressure force on the projected area | N |
| $F_W$ | force carried by one wall section | N |
| $A_{proj}$ | projected area of the element | $m^2$ |
| $\sigma_{zul}$ | permissible stress in the wall | Pa |
| $s$ | wall thickness | m |
| $L$ | length of the vessel | m |

For a pipe with inside diameter $d$, $A_{proj} = d \cdot L$, and the pressure force is shared by two wall sections: $F_z = 2\,F_W$. This gives the required wall thickness $s = p \cdot d / (2\,\sigma_{zul})$.

If an open vessel is accelerated horizontally, the surface tilts; if it rotates, the surface becomes a paraboloid.

$$\tan\alpha = \frac{a}{g} \qquad z = \frac{\omega^2}{2g} \cdot r^2 + Z_s \tag{3.6}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $a$ | acceleration of the vessel | $m\,s^{-2}$ |
| $\omega$ | angular velocity | $s^{-1}$ |
| $Z_s$ | vertex height, lowest liquid level | m |
| $r$ | radius, measured from the axis | m |

Equality of volume between rest and rotation gives the result that the liquid rises at the wall of a vessel with radius $R$ by the same amount $h_1$ as it falls at the centre ($h_2$): $h_1 = h_2 = \omega^2 R^2 / (4g)$.

# 4 Continuity and Bernoulli

Two laws carry steady flow calculation: the continuity equation for mass, Bernoulli's equation for energy.

## 4.1 Volume and mass flow rate

$$\dot{V} = A \cdot \bar{c} \qquad \dot{m} = \rho \cdot \dot{V} \tag{4.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\dot{V}$ | volume flow rate | $m^3\,s^{-1}$ |
| $\dot{m}$ | mass flow rate | $kg\,s^{-1}$ |
| $A$ | cross-sectional area carrying the flow | $m^2$ |
| $\bar{c}$ | mean velocity over the cross section | $m\,s^{-1}$ |

For a velocity that is not uniform across the section, the integral form applies. For a circular section the one-dimensional form is sufficient.

$$\dot{V} = \int_A c(r)\,\mathrm{d}A = 2\pi \int_0^R c(r) \cdot r\,\mathrm{d}r$$

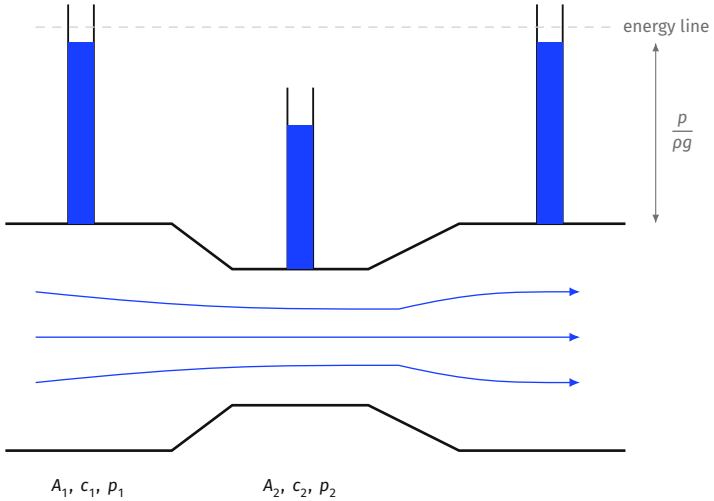
## 4.2 Continuity equation

$$\sum_i \dot{m}_i = 0 \qquad \text{hence} \qquad \rho_1 c_1 A_1 = \rho_2 c_2 A_2 \tag{4.2}$$

$$\dot{V}_1 = \dot{V}_2 \iff c_1 A_1 = c_2 A_2 \qquad \text{(incompressible)}$$

The simple form holds as long as density does not change. For gases it holds up to around 5 % change in pressure; beyond that, densities have to be carried through.

### 4.3 Bernoulli equation without losses




At the restriction the velocity rises and the static pressure falls. The sum of static pressure head, geodetic head and velocity head stays constant: the energy line.

Three ways of writing the same statement. The choice follows the unit the result is wanted in: specific energy, pressure or head.

$$\frac{p}{\rho} + g \cdot z + \frac{c^2}{2} = \text{const.} \tag{4.3}$$

$$p + \rho \cdot g \cdot z + \frac{\rho}{2} \cdot c^2 = \text{const.} \tag{4.4}$$

$$\frac{p}{\rho \cdot g} + z + \frac{c^2}{2g} = \text{const.} \tag{4.5}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p$ | static pressure | Pa |
| $\rho g z$ | geodetic pressure | Pa |
| $\frac{\rho}{2} c^2$ | dynamic pressure | Pa |
| $z$ | geodetic height | m |

### 4.4 Bernoulli equation with losses

Real flows lose energy to friction and to the formation of eddies. The loss term sits on the side of the second point: by then it has been spent.

$$\frac{p_1}{\rho} + g z_1 + \frac{c_1^2}{2} = \frac{p_2}{\rho} + g z_2 + \frac{c_2^2}{2} + Y_{V_{1-2}} \tag{4.6}$$

$$p_1 + \rho g z_1 + \frac{\rho}{2} c_1^2 = p_2 + \rho g z_2 + \frac{\rho}{2} c_2^2 + p_{V_{1-2}} \tag{4.7}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $Y_{V_{1-2}}$ | specific loss energy | J kg$^{-1}$ |
| $p_{V_{1-2}}$ | pressure loss | Pa |
| $H_{V_{1-2}}$ | head loss | m |

The three loss quantities are one quantity in three units: $Y_V = p_V/\rho = H_V \cdot g$.

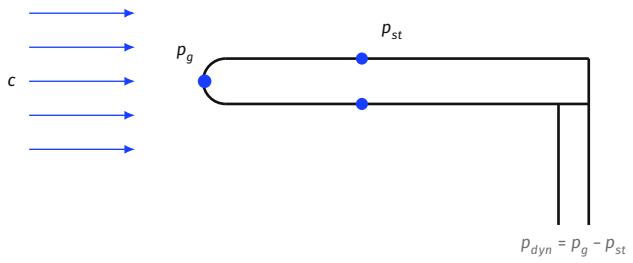
**VALIDITY OF THE BERNOULLI EQUATION**

Strictly, Bernoulli's equation holds along *one* streamline in the steady flow of a frictionless fluid of constant density; for gases it holds, like the continuity equation, up to around 5 % change in pressure. It holds across the whole cross section only once the streamlines run parallel and the velocity is uniformly distributed.

Hence the inlet runs specified in EN ISO 5167. A bend upstream of the measuring point disturbs the velocity profile and so violates the premise of the evaluation. Under EN ISO 5167-4 the classical Venturi tube needs the shortest inlet runs, from 2.5 × *D* depending on the upstream fitting and the diameter ratio.

## 5 Consequences of the Bernoulli equation

### 5.1 Pitot-static tube




The bore at the tip picks up total pressure, the bores in the casing the static pressure. Their difference is the dynamic pressure.

At the stagnation point the flow comes to rest. The velocity energy is converted entirely into pressure.

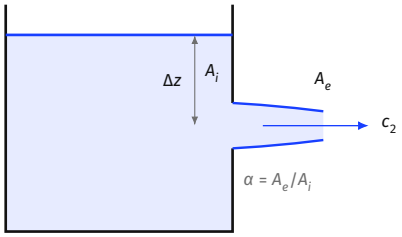
$$p_g = p_{st} + p_{dyn} \qquad \text{with} \qquad p_{dyn} = \frac{\rho}{2} \cdot c^2 \tag{5.1}$$

$$c = \sqrt{\frac{2 \cdot (p_g - p_{st})}{\rho}}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p_g$ | total pressure | Pa |
| $p_{st}$ | static pressure | Pa |
| $p_{dyn}$ | dynamic pressure | Pa |

Density sits under the root. An assumed rather than measured density carries half of its error into the result; in gases this is the largest single contribution to measurement uncertainty.

## 5.2 Torricelli discharge equation




The emerging jet is narrower than the opening. The ratio of the two areas is the contraction coefficient.

$$c_2 = \sqrt{2 \cdot (g \cdot \Delta z - Y_{V_{1-2}})} \tag{5.2}$$

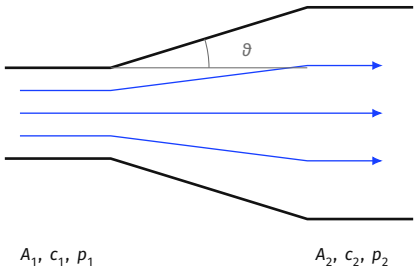
$$\alpha = \frac{A_e}{A_i} \tag{5.3}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\Delta z$ | height difference between surface and opening | m |
| $\alpha$ | contraction coefficient | – |
| $A_e$ | cross section of the jet | $m^2$ |
| $A_i$ | cross section of the opening | $m^2$ |

| FORM OF THE OPENING | $\alpha$ |
|---|---|
| **sharp-edged hole, circular** | 0.58 |
| **sharp-edged hole, rectangular** | 0.61 |
| **Borda mouthpiece, sharp-edged** | 0.50 |
| **rounded nozzle** | ≈ 1 |

Holds for a vessel cross section large enough to make the rate at which the surface drops negligible.

### 5.3 Diffuser and confuser




The diffuser decelerates the flow and recovers static pressure. At too wide an opening angle the flow separates and the gain fails to appear.

$$p_2' = p_1 + \rho \cdot \frac{c_1^2}{2} \cdot \left[1 - \left(\frac{A_1}{A_2}\right)^2\right] \tag{5.4}$$

$$\eta_D = \frac{p_2 - p_1}{p_2' - p_1} \tag{5.5}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $p_2'$ | theoretical pressure without losses | Pa |
| $p_2$ | actual pressure after the diffuser | Pa |
| $\eta_D$ | diffuser efficiency | – |

A diffuser with a small opening angle reaches $\eta_D \approx 0.95$. The pressure loss is then $p_v = (1 - \eta_D) \cdot \frac{\rho}{2} c_1^2 \left[1 - (A_1/A_2)^2\right]$.

**THE DIFFUSER DECIDES THE PRESSURE LOSS**

Venturi tubes and Venturi nozzles have a diffuser, orifice plates do not. The permanent pressure loss hangs on that: the DV700 Venturi nozzle gives up 10 to 15 % of the differential pressure, the MBL500 orifice measuring section 30 to 80 %. In continuous operation this is a question of operating cost.

### 5.4 Pressure change normal to the streamline

On a curved path the fluid needs a centripetal force; it comes from a pressure gradient normal to the flow. A tapping for static pressure therefore does not belong downstream of a bend.

$$\mathrm{d}p_n = \rho \cdot \frac{\bar{c}^2}{r} \cdot \mathrm{d}n \tag{5.6}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\mathrm{d}p_n$ | pressure change normal to the flow | Pa |
| $\mathrm{d}n$ | path element normal to the streamline, counted outward from the centre of curvature | m |
| $r$ | mean radius of curvature | m |

In a straight section carrying flow the static pressure is constant across the area, even with separated flow and an uneven velocity distribution. Only curvature changes that.

## 6 Momentum equation

The momentum equation gives the force a flowing fluid exerts on the component it flows through. It needs only inlet and outlet, no assumption about what happens inside.

$$\sum_i \vec{F}_{Ii} = \dot{m} \cdot \left(\vec{c}_1 - \vec{c}_2\right) \tag{6.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\vec{F}_{Ii}$ | momentum force | N |
| $\vec{c}_1$ | velocity at the inlet | $\mathrm{m\,s^{-1}}$ |
| $\vec{c}_2$ | velocity at the outlet | $\mathrm{m\,s^{-1}}$ |

A vector equation; signs follow the chosen coordinate system. Pressure and momentum forces always point into the control volume. With several inlets or outlets, $\dot{m} \cdot \vec{c}$ is replaced by the sum of the $\dot{m}_i \cdot \vec{c}_i$.

In the steady case momentum forces and external forces $\vec{F}_{Sk}$ on the control volume balance. In this form restraining forces at pipe bends and test fixtures are calculated.

$$\dot{m} \cdot \left(\vec{c}_2 - \vec{c}_1\right) = \sum_k \vec{F}_{Sk} \tag{6.2}$$

A free jet striking a fixed wall at an angle divides. Force and division follow from the momentum and continuity equations.

$$F_W = \rho \cdot A_D \cdot c_D^2 \cdot \sin\delta \tag{6.3}$$

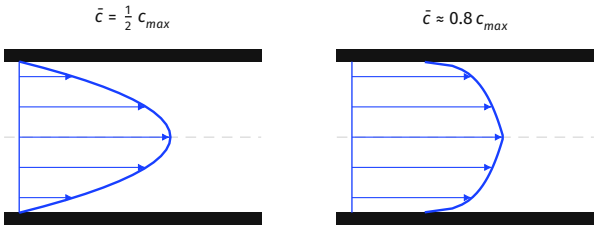
$$\dot{V}_2 = \frac{1}{2}\dot{V}_1(1 - \cos\delta) \qquad \dot{V}_3 = \frac{1}{2}\dot{V}_1(1 + \cos\delta)$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $F_W$ | force of the jet on the wall, normal to it | N |
| $A_D$ | cross section of the jet | $m^2$ |
| $c_D$ | velocity of the jet | $m\,s^{-1}$ |
| $\delta$ | angle between jet and wall | ° |
| $\dot{V}_1$ | volume flow rate of the jet | $m^3\,s^{-1}$ |
| $\dot{V}_2$ | partial flow along the wall, against the jet direction | $m^3\,s^{-1}$ |
| $\dot{V}_3$ | partial flow along the wall, in the jet direction | $m^3\,s^{-1}$ |

At $\delta = 90°$ the jet divides in equal parts: $\dot{V}_2 = \dot{V}_3 = \frac{1}{2}\dot{V}_1$.

# 7 Pipe flow

## 7.1 Reynolds number




On the left the parabolic profile of laminar flow, on the right the fuller profile of turbulent flow. The ratio of mean to maximum velocity rises from 0.5 to around 0.8.

The Reynolds number is the dimensionless ratio of inertial to viscous forces. It decides whether a flow stays laminar.

$$\mathrm{Re} = \bar{c} \cdot \frac{D}{\nu} \qquad \mathrm{Re} = c_{\infty} \cdot \frac{L}{\nu} \tag{7.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| Re | Reynolds number | – |
| $D$ | pipe diameter | m |
| $L$ | characteristic length (plate, chord) | m |
| $c_{\infty}$ | free-stream velocity | $\mathrm{m\,s^{-1}}$ |

| | |
|---|---|
| Re < 2320 | laminar flow – ordered layers, no transverse exchange |
| Re > 2320 | turbulent flow – transverse exchange, fuller profile, higher loss |

### THE LAMINAR FLOW ELEMENT KEEPS REYNOLDS LOW

A laminar flow element divides the total flow among many narrow channels. This keeps the Reynolds number in each single channel below the transition value, the flow stays laminar, and the Hagen-Poiseuille law applies: the volume flow rate is *proportional* to the differential pressure; at an orifice plate or nozzle it rises only with its square root. Hence the wide span of these instruments – the S series reaches up to 1:50 with the glass capillaries of the S010 and 1:20 with a stainless steel matrix, the R series 1:10 and up to 1:30 in special cases.

The price of that linearity is a dependence on viscosity: in the Hagen-Poiseuille relationship $\eta$ sits in the denominator, and $\eta$ depends on gas type and temperature. Our evaluation therefore carries density, viscosity and Reynolds number through the calculation.

### 7.2 Laminar pipe flow

At Re < 2320 the velocity profile is a parabola and the mean velocity half the maximum. For this case the Hagen-Poiseuille law gives the volume flow rate in closed form.

$$c(r) = \frac{\Delta p}{4 \cdot \eta \cdot L} \cdot \left(R^2 - r^2\right) \tag{7.2}$$

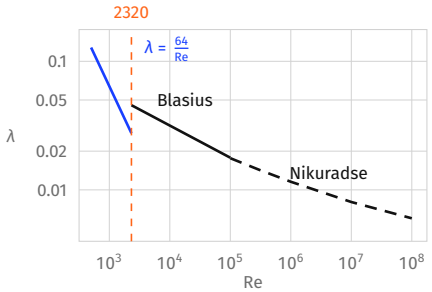
$$c_{max} = \frac{\Delta p}{4 \cdot \eta \cdot L} \cdot R^2 \qquad \bar{c} = \frac{1}{2} \cdot c_{max}$$

$$\dot{V} = \frac{\pi \cdot \Delta p \cdot R^4}{8 \cdot \eta \cdot L} \tag{7.3}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\Delta p$ | static pressure difference over the length $L$ | Pa |
| $L$ | length of the pipe section | m |
| $R$ | inside radius of the pipe | m |
| $r$ | running variable across the section | m |

The radius enters in the fourth power: 10 % less radius gives 35 % less volume flow rate. In a capillary calibrated leak the manufacturing tolerance of the diameter therefore governs the leak rate.

### 7.3 Pipe friction factor




The pipe friction factor against the Reynolds number, on logarithmic scales. At 2320 it jumps: the transition from the laminar straight line to the turbulent law is discontinuous.

The pressure loss of a length of pipe follows from the friction factor. In the laminar region it depends on the Reynolds number alone, in the turbulent region on wall roughness as well.

$$Y_V = \lambda \cdot \frac{L}{D} \cdot \frac{\bar{c}^2}{2} \qquad \zeta = \lambda \cdot \frac{L}{D} \tag{7.4}$$

$$\lambda = \frac{64}{\mathrm{Re}} \qquad (\mathrm{Re} < 2320) \tag{7.5}$$

$$\lambda = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}} \qquad (2320 < \mathrm{Re} < 10^5) \tag{7.6}$$

$$\lambda = 0.0032 + 0.221 \cdot \mathrm{Re}^{-0.237} \qquad (10^5 < \mathrm{Re} < 10^8) \tag{7.7}$$

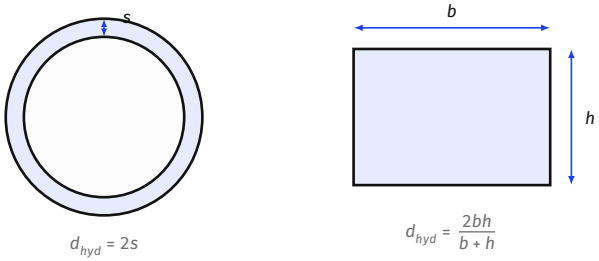
$$\frac{1}{\sqrt{\lambda}} = 2 \cdot \log\left(\mathrm{Re} \cdot \sqrt{\lambda}\right) - 0.8 \qquad (\mathrm{Re} > 2320) \tag{7.8}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\lambda$ | pipe friction factor | – |
| $\zeta$ | loss coefficient of the section | – |
| $k_s$ | equivalent sand roughness of the wall | m |

| RANGE OF VALIDITY | LAW |
|---|---|
| $\mathrm{Re} < 2320$ | $\lambda = 64/\mathrm{Re}$ – exact, derived from Hagen-Poiseuille |
| $2320 < \mathrm{Re} < 10^5$, **smooth** | Blasius |
| $10^5 < \mathrm{Re} < 10^8$, **smooth** | Nikuradse |
| $\mathrm{Re} > 2320$, **smooth, universal** | Prandtl-Kármán, to be solved iteratively |
| $25 < \frac{k_s}{D}\mathrm{Re}^{0.875} < 350$ | transition region, Prandtl-Colebrook, to be solved iteratively: $\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2.51}{\sqrt{\lambda}\cdot\mathrm{Re}} + 0.27\frac{k_s}{D}\right)$ |
| $\frac{k_s}{D}\mathrm{Re}^{0.875} \geq 350$ | fully rough: $\lambda = \left[1.14 - 2\log\frac{k_s}{D}\right]^{-2}$ – independent of Re |

In an annulus, $\lambda = 0.2236/\sqrt[4]{\mathrm{Re}}$ replaces Blasius. The shear stress at the pipe wall is $\tau_0 = \frac{\lambda}{8} \cdot \rho \cdot \bar{c}^2$.

### 7.4 Hydraulic diameter




For a non-circular cross section the hydraulic diameter takes the place of the pipe diameter. What governs is the wetted perimeter; in an annulus it counts inside and outside.

$$d_{hyd} = \frac{4 \cdot A_{UR}}{U_{ges}} \tag{7.9}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $A_{UR}$ | area of the non-circular section carrying flow | $m^2$ |
| $U_{ges}$ | wetted perimeter, inside and outside where applicable | m |

| CROSS SECTION | HYDRAULIC DIAMETER |
|---|---|
| **annulus of gap width** $s$ | $2s$ |
| **closed rectangular duct** $b \times h$ | $\frac{2bh}{b+h}$ |
| **open rectangular channel** | $\frac{2bh}{h+0.5\,b}$ |

## 8 Pressure losses at fittings

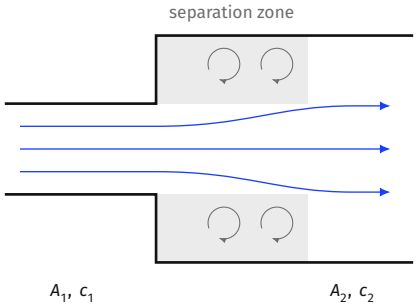
Every fitting costs energy. The loss coefficient $\zeta$ gives the energy converted into heat, referred to the kinetic energy $c^2/2$.

$$Y_{V_{1\text{-}2}} = \zeta \cdot \frac{c^2}{2} \tag{8.1}$$

| SYMBOL | MEANING | UNIT |
|---|---|---|
| $\zeta$ | loss coefficient | – |
| $c$ | reference velocity – *which* one is part of the figure | $m\,s^{-1}$ |

A loss coefficient holds only together with its reference velocity. At a change of cross section the values for $c_1$ and $c_2$ differ by the square of the area ratio.

### 8.1 Sudden enlargement




Downstream of the step the flow separates. The loss arises in the eddies of the separation zone.

$$\zeta_1 = \left(1 - \frac{A_1}{A_2}\right)^2 \qquad \zeta_2 = \left(\frac{A_2}{A_1} - 1\right)^2 \tag{8.2}$$

$\zeta_1$ refers to $c_1$, $\zeta_2$ to $c_2$. In the limiting case $A_1 \ll A_2$ – discharge into a large space – $\zeta_{out} = 1$: the entire kinetic energy is lost.

### 8.2 Sudden contraction

The loss arises from contraction and the re-expansion behind the edge. That is why the contraction coefficient appears in the formula.

$$\zeta_1 = \left(\frac{1}{\alpha} - 1\right)^2 \cdot \left(\frac{A_1}{A_2}\right)^2 \qquad \zeta_2 = \left(\frac{1}{\alpha} - 1\right)^2 \tag{8.3}$$

In the limiting case $A_1 \gg A_2$ – entry from a large space – $\alpha = 0.58$ and hence $\zeta_{in} = 0.52 \approx 0.5$. Exactly, $\alpha = A_3/A_2$ with $A_3$ the narrowest jet cross section; since $A_3$ is not accessible, $\alpha$ is determined experimentally against $A_2/A_1$.

**PRESSURE DROP AS THE MEASURED QUANTITY**

In leak testing the pressure drop is the measurement signal. It has two causes: leakage and thermal effects from the heat of compression during filling. Settling time and a reference volume separate them.

## 9 Units, constants, conversions

| QUANTITY | CONVERSION |
|---|---|
| **Pressure** | 1 bar = $10^5$ Pa = $10^5$ N/m$^2$ = 0.1 N/mm$^2$; 1 mbar = 100 Pa |
| **Force** | 1 N = 1 kg · m · s$^{-2}$ |
| **Energy, work** | 1 J = 1 N · m = 1 kg · m$^2$ · s$^{-2}$ = 1 W · s |
| **Power** | 1 W = 1 J · s$^{-1}$ = 1 N · m · s$^{-1}$ |
| **Acceleration** | 1 m · s$^{-2}$ = 1 N · kg$^{-1}$ |
| **Volume flow rate** | 1 m$^3$/h = 16.$\overline{6}$ L/min = 277.$\overline{7}$ cm$^3$/s |
| **Angular velocity** | $\omega = 2\pi n$; as a numerical equation $n = \frac{\omega}{2\pi} \cdot 60$ in min$^{-1}$ for $\omega$ in s$^{-1}$ |

## Material values used in this collection

| QUANTITY | VALUE | APPLIES TO |
|---|---|---|
| **Specific gas constant** $R_i$ | 287.05 J kg$^{-1}$ K$^{-1}$ | dry air, ISO 2533 |
| **Isentropic exponent** $\kappa$ | 1.4 | air, diatomic |
| **Gravitational acceleration** $g$ | 9.81 m s$^{-2}$ | rounded; standard value 9.80665 |
| **Air density** $\rho_{b0}$ | 1.225 kg m$^{-3}$ | ICAO, sea level |
| **Air pressure** $p_{b0}$ | 1013.25 mbar | ICAO, sea level |
| **Temperature** $T_{b0}$ | 288.15 K | ICAO, sea level |
| **Transition value** | Re = 2320 | pipe flow |

For a calculation with a stated measurement uncertainty, current material data for the gas in question should be used.

## Where this physics is put to work here

### Calibration laboratory

DAkkS and factory calibrations for quantities of gas flow measurement, in the laboratory or on site. Accredited to EN ISO/IEC 17025 since 1999, registration no. D-K-17589-01-00.

The accreditation certificate lists three gas ranges: $N_2$, $H_2$, $CH_4$, He and Ar from 10 L/h to 9 $m^3$/h, CO from 10 L/h to 1.5 $m^3$/h, propane from 0.5 L/h to 600 L/h. On-site calibration forms a section of its own in the certificate.

### Measurement and test technology

Sensors and systems for air and gas flow, pressure, temperature, humidity and leak tightness – laminar flow elements, differential pressure devices to EN ISO 5167, capillary calibrated leaks and complete test rigs.

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## About this document

| | |
|---|---|
| **Issue** | Version 1.0, 2026-11-12 |
| **Text and figures** | Own work of TetraTec Instruments GmbH |

The equations are taken from a German formula collection on applied fluid mechanics dated 07 March 2002, recalculated and corrected where necessary.

This collection replaces neither a textbook nor a sizing calculation. It reflects our present knowledge and serves as orientation; no liability for completeness or accuracy is accepted. What governs the sizing of a measuring set-up are the applicable standards and the data of the instrument used. For a particular case, please talk to us.