Laminar Flow Elements

Technical Paper

A laminar flow element measures gas flow through a differential pressure that is nearly proportional to the volumetric flow. This paper shows where the proportionality comes from and how calibration and evaluation deal with its limits.

The nine chapters

- **Flow quantities** – differential pressure, continuity
- **Fundamentals** – viscosity, pipe flow, vortices
- **Measuring principle** – designs, Hagenbach, turn-down
- **Calibration** – linear and universal characteristic
- **Evaluation** – from differential pressure to flow
- **Conversion** – actual, standard and mass flow
- **Gas properties** – density, viscosity, sensors
- **Application** – testing cross-sections
- **Summary**

How to use it

Every equation sits in a grey box and carries its number on the right. Below it, a table lists each new symbol with meaning and unit.

Derivations stand between the equations without a box. If you only need the result, you can skip them.

The blue boxes name where the relationship acts in our instruments. The diagrams in chapter 4 show measured values of real LFEs.

WHY WE PUBLISH THIS

An LFE measures as accurately as it is evaluated.

We build, calibrate and operate laminar flow elements. This is the physics behind them, including its limits.

Symbols and conventions

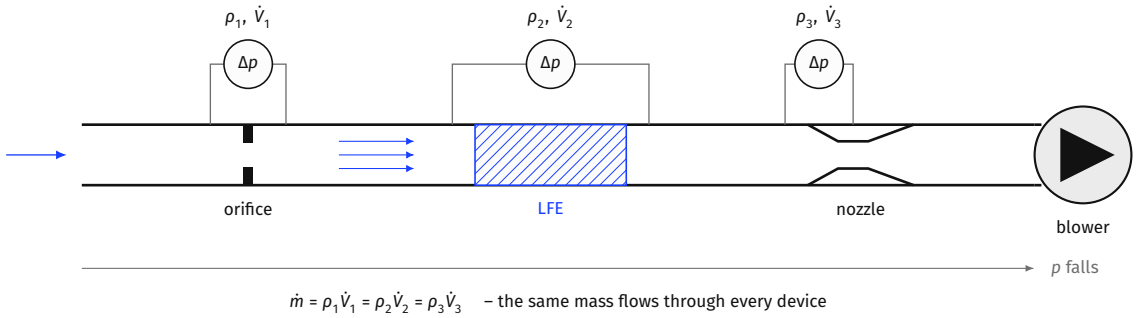
System of units	SI throughout. Pressures are **absolute pressures** unless stated otherwise.
Volumetric flow	$\dot{V}$ without index is the actual volumetric flow at the LFE, in the state upstream of the element.
Indices	1 and 2: upstream and downstream of the LFE, in chapter 8 of the test piece. n: standard conditions. kal: calibration conditions.
Former symbols	Our older documents write Q_{vac}, Q_{vno} and Q_m for $\dot{V}$, $\dot{V}_n$ and $\dot{m}$.

List of symbols

A	area, cross-section
a	speed of sound
a_1, a_2	coefficients of the Hagen-Poiseuille characteristic
$b_1 \dots b_3$	coefficients of the universal characteristic
$C_1 \dots C_3$	constants of the LFE
C_D	discharge coefficient
c	velocity
$\bar{c}$	mean velocity
c_w	drag coefficient
d, d_{hyd}	diameter, hydraulic diameter
F	force
g	gravitational acceleration
h	plate spacing
L	length
M	molar mass
$\dot{m}$	mass flow
N	number of parallel channels
P	power
p, Δp	pressure, differential pressure
p_s	saturation vapour pressure
Po	Poiseuille number
R, r	radius
R_i	specific gas constant
Re	Reynolds number
s	distance, gap width
T	absolute temperature
$\dot{V}$	volumetric flow
W	work, potential
X, Y	axes of the universal characteristic
x_v	mole fraction of water vapour
Z	compressibility factor
z	coordinate across the flow
η	dynamic viscosity
ϑ	temperature in °C
κ	isentropic exponent
λ	pipe friction factor
ν	kinematic viscosity
ρ	density
τ	shear stress
φ	relative humidity

1 Flow quantities and differential pressure

Differential pressure devices measure the pressure difference that a primary element creates in the flow and infer the flow rate from it. Their advantage over other methods is the short response time: the differential pressure follows a change in flow practically at once.



Three differential pressure devices in one line, driven by a blower. The pressure falls from device to device. The mass flowing through each is the same; the volume changes with the density.

1.1 Conservation of mass

Gases are compressible: the same mass occupies a different volume at a different pressure and temperature. The continuity equation states what stays the same along a line.

$$\dot{m} = \rho_1 \cdot \dot{V}_1 = \rho_2 \cdot \dot{V}_2 = \rho_n \cdot \dot{V}_n \tag{1.1}$$

SYMBOL	MEANING	UNIT
$\dot{m}$	mass flow	$kg\,s^{-1}$
$\dot{V}_1, \dot{V}_2$	actual volumetric flow at point 1 or 2	$m^3\,s^{-1}$
ρ_1, ρ_2	density in the respective operating state	$kg\,m^{-3}$
$\dot{V}_n$	standard volumetric flow	$m^3\,s^{-1}$
ρ_n	density at standard conditions	$kg\,m^{-3}$

A volumetric flow is only fully specified together with its state. Chapter 6 gives the conversion between the quantities.

1.2 The LFE as a linear differential pressure device

Orifice, nozzle and Venturi tube produce a differential pressure that grows with the square of the flow. The laminar flow element, LFE for short, guides the flow through many narrow channels in which it stays laminar. Its differential pressure depends mainly on the viscosity and grows almost linearly with the volumetric flow. The LFE thus combines the speed of the differential pressure method with a much larger turndown (section 3.7).

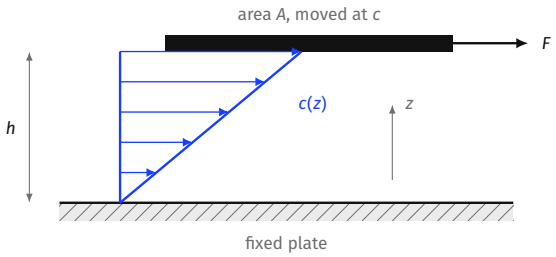
EVERY LFE IS SUPPLIED CALIBRATED

The narrow channels can only be made to tolerances that amount to about ±10 % in flow. Section 2.3 shows why so much: the radius enters in the fourth power. We therefore supply LFEs for flow measurement calibrated only.

2 Physical fundamentals

2.1 Viscous friction

In a real fluid the particles do not move independently of each other. When layers slide past one another, internal friction arises; macroscopically it shows as viscosity.



Arrangement for measuring viscous friction. The lower plate is at rest, the upper plate of area A is pulled at velocity c. The velocity profile in between is linear to a good approximation.

The experiment shows: the force grows with the area and with the velocity and falls with the spacing of the plates.

$$F \sim A, \quad F \sim c, \quad F \sim \frac{1}{h} \quad \Longrightarrow \quad F \sim A \cdot \frac{c}{h} \tag{2.1}$$

A material constant turns this into an equation. The constant η is called *dynamic viscosity*.

$$F = \eta \cdot A \cdot \frac{c}{h} \tag{2.2}$$

SYMBOL	MEANING	UNIT
F	force on the moving plate	N
A	area of the moving plate	m^2
c	velocity of the plate	$m\,s^{-1}$
h	spacing of the plates	m
η	dynamic viscosity	$Pa\,s = N\,s\,m^{-2}$

If the profile is not linear, the relationship holds locally, for every thin layer. This is Newton's law of friction; it applies to any velocity distribution.

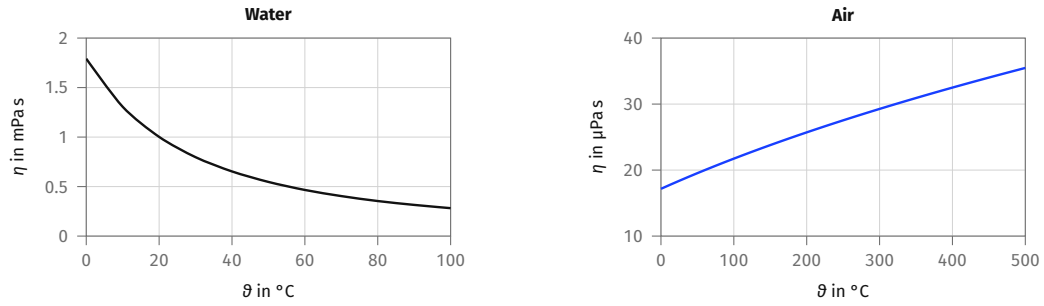
$$F = \eta \cdot A \cdot \frac{dc}{dz} \tag{2.3}$$

Referred to the area, friction is a shearing process: η links the shear stress with the shear rate.

$$\tau = \frac{F}{A} = \eta \cdot \frac{dc}{dz} \tag{2.4}$$

SYMBOL	MEANING	UNIT
τ	shear stress	Pa
dc/dz	shear rate	s^{-1}
z	coordinate across the flow	m

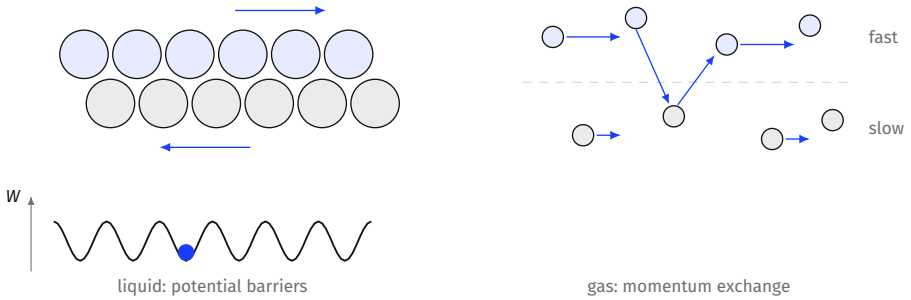
2.2 Viscosity and temperature



Dynamic viscosity versus temperature at atmospheric pressure. Water: tabulated values. Air: calculated after Sutherland, equation (7.3).

Liquids and gases behave in opposite ways. In a liquid the molecules are closely packed. Shifting two layers against each other means overcoming potential barriers, and thermal motion helps: η falls with rising temperature.

In a gas the molecules move freely. Friction arises because they change between fast and slow layers and carry momentum with them. More thermal motion means more exchange: in gases η rises with temperature.



Left: two molecular layers of a liquid and the potential W that every displacement has to overcome. Right: a gas, in which molecules change between fast and slow layers.

2.3 Flow in a pipe: Hagen-Poiseuille

The law of friction allows the laminar flow in a pipe to be calculated. The driving force is the pressure difference between the pipe ends. On a fluid cylinder of radius r it balances the shear stress on the cylinder surface. Integrated with the no-slip condition $c(R) = 0$ – the fluid is at rest at the wall –, this gives the velocity profile.

$$\Delta p \cdot \pi r^2 = -\eta \cdot \frac{\mathrm{d}c}{\mathrm{d}r} \cdot 2\pi r L \qquad \Longrightarrow \qquad c(r) = \frac{\Delta p}{4\,\eta\,L} \cdot \left(R^2 - r^2\right)$$

Integrated over the cross-section, the parabolic profile gives the volumetric flow, the law of Hagen-Poiseuille.

$$\dot{V} = \int_0^R c(r) \cdot 2\pi r \,\mathrm{d}r = \frac{\pi\,\Delta p}{2\,\eta\,L} \int_0^R \left(R^2 r - r^3\right) \mathrm{d}r = \frac{\pi\,\Delta p}{2\,\eta\,L} \cdot \frac{R^4}{4}$$

$$\dot{V} = \frac{\pi \cdot R^4}{8 \cdot \eta \cdot L} \cdot \Delta p \tag{2.5}$$

SYMBOL	MEANING	UNIT
$\dot{V}$	volumetric flow	$\mathrm{m^3\,s^{-1}}$
Δp	pressure difference $p_1 - p_2$ over the length L	Pa
R	inner radius of the pipe	m
L	length of the pipe	m
r	radius as running variable	m
$c(r)$	local velocity	$\mathrm{m\,s^{-1}}$

The volumetric flow is proportional to the pressure difference and inversely proportional to the viscosity. The LFE rests on this law. The radius appears in the fourth power; a small deviation δR in radius therefore acts fourfold on the volumetric flow:

$$\frac{\delta \dot{V}}{\dot{V}} = 4 \cdot \frac{\delta R}{R}$$

A deviation of 2.5 % in radius gives 10 % in volumetric flow.

2.4 Flow around a sphere: Stokes

Stokes' law gives the friction force on a sphere of radius r in slow flow.

$$F = 6\pi \cdot \eta \cdot r \cdot c \tag{2.6}$$

The falling-ball viscometer is based on it. A sphere sinks in a fluid at rest and speeds up until friction and buoyancy carry its weight; from then on it sinks at the constant velocity c. With the sphere volume $\frac{4}{3}\pi r^3$:

$$\frac{4}{3}\pi r^3 \cdot (\rho_K - \rho_F) \cdot g = 6\pi \cdot \eta \cdot r \cdot c$$

$$c = \frac{2\,r^2\,g \cdot (\rho_K - \rho_F)}{9\,\eta} \tag{2.7}$$

The *kinematic viscosity* is the dynamic viscosity referred to the density of the fluid.

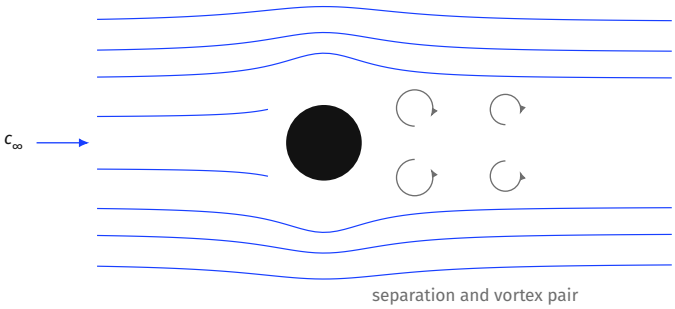
$$v = \frac{\eta}{\rho_F} = \frac{2\,r^2\,g}{9\,c} \cdot \frac{\rho_K - \rho_F}{\rho_F} \tag{2.8}$$

SYMBOL	MEANING	UNIT
r	radius of the sphere	m
c	sinking velocity	$\mathrm{m\,s^{-1}}$
ρ_K	density of the sphere	$\mathrm{kg\,m^{-3}}$
ρ_F	density of the fluid	$\mathrm{kg\,m^{-3}}$
g	gravitational acceleration, 9.81	$\mathrm{m\,s^{-2}}$
v	kinematic viscosity	$\mathrm{m^2\,s^{-1}}$

Stokes' law applies to creeping flow: the Reynolds number (2.9), formed with the sphere diameter, is below 1.

A comparison that surprises: air at 20 °C has a kinematic viscosity of about $15 \cdot 10^{-6}\,\mathrm{m^2/s}$, water $1.0 \cdot 10^{-6}\,\mathrm{m^2/s}$. Kinematically, air is about fifteen times more viscous than water; its low density more than outweighs its low dynamic viscosity.

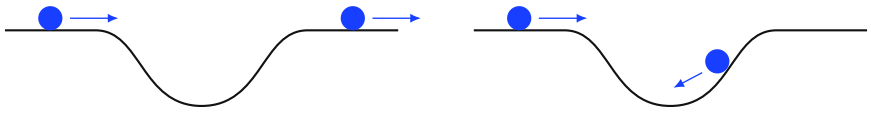
2.5 Vortex formation and Reynolds number



Flow around a cylinder. Behind the body the flow separates, and two vortices of opposite sense of rotation form.

At higher flow velocities, vortices form behind an obstacle; as the velocity rises further, the flow becomes turbulent. The vortices form in pairs with opposite sense of rotation, and their angular momentum cancels.

In an ideal, frictionless fluid the flow accelerates around the obstacle as in a pipe constriction and is slowed behind it to its initial velocity again. A fluid particle behaves like a ball rolling through a hollow: without friction it leaves the hollow as fast as it entered.



Analogy for flow separation. With friction the energy of the ball is not sufficient for the climb, and it turns back.

With friction, particles near the wall lose energy. Behind the obstacle they run against the rising pressure, come to rest and turn back. The streamlines separate from the body and roll up. An unfavourable shape strengthens the effect: if the body accelerates the flow strongly as it passes, a “deep hollow”, separation sets in at a lower velocity. The vortices draw kinetic energy from the flow, which further internal friction turns into heat.

Whether it comes to this is decided by the ratio of two forces. Inertial forces grow with the square of the velocity, friction forces only linearly. Their ratio is the Reynolds number.

$$\mathrm{Re} = \frac{\rho \cdot c \cdot d}{\eta} = \frac{c \cdot d}{\nu} \tag{2.9}$$

SYMBOL	MEANING	UNIT
Re	Reynolds number	–
ρ	density of the fluid	kg m^{-3}
d	characteristic length: pipe, channel or sphere diameter	m
c	velocity, in a pipe the mean velocity	m s^{-1}

In a straight pipe the flow stays laminar below a Reynolds number of about 2300.

2.6 Flow resistance

Without vortices and without friction an obstacle would offer no resistance. The dynamic pressure that builds up at the expense of the static pressure as the flow passes the sides would be fully converted back on the rear, and no energy would be taken from the flow. With vortex formation this conversion succeeds only in part; part of the pressure is missing on the rear. The drag force is therefore proportional to the dynamic pressure.

$$F = c_w \cdot A \cdot \frac{\rho}{2} c^2 \tag{2.10}$$

SYMBOL	MEANING	UNIT
c_w	drag coefficient	–
A	cross-sectional area of the body	m^2
$\frac{\rho}{2}c^2$	dynamic pressure	Pa

The coefficient depends strongly on the shape of the body, because the shape determines the vortex formation; for a car it lies between 0.3 and 0.5. For the friction work over the distance s and the power loss:

$$W = F \cdot s = c_w \cdot A \cdot \frac{\rho}{2} c^2 \cdot s \qquad P = F \cdot c = c_w \cdot A \cdot \frac{\rho}{2} c^3 \tag{2.11}$$

SYMBOL	MEANING	UNIT
W	friction work over the distance s	J
s	distance covered	m
P	power loss	W

In creeping flow the force is proportional to the velocity (2.6), with vortex formation to its square (2.10). Equating the two, with the frontal area $A = \pi r^2$ and the diameter $d = 2r$, shows how the drag coefficient of the sphere depends on the Reynolds number in the Stokes regime:

$$c_w = \frac{6\pi \cdot \eta \cdot r \cdot c}{\pi r^2 \cdot \frac{\rho}{2} c^2} = \frac{12\,\eta}{\rho \cdot c \cdot r} = \frac{24\,\eta}{\rho \cdot c \cdot d} = \frac{24}{\mathrm{Re}} \qquad (\mathrm{Re} < 1) \tag{2.12}$$

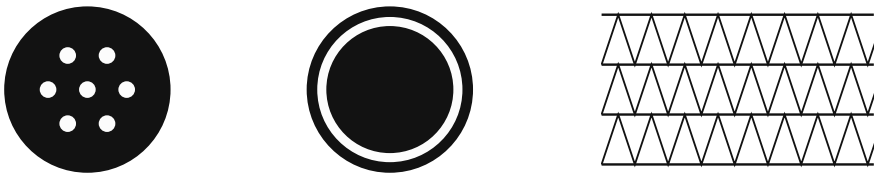
TWO LAWS IN ONE ELEMENT

Linear and quadratic resistance side by side recur in the LFE: the friction part of its differential pressure grows with the volumetric flow, the acceleration part with its square (equation 3.5). An LFE is designed so that the linear part dominates.

3 Measuring principle of the laminar flow element

3.1 Designs

Laminar flow elements classically consist of one or more capillaries with a very small diameter, below 1 mm, and lengths of 80 to 200 mm, or of channels formed by very narrow annular gaps. For larger flows the capillaries are bundled. For large flows, a wound pair of corrugated and flat stainless steel strip forms a matrix of parallel triangular channels, fitted as a coil into a housing.



capillary bundle | annular gap | matrix, detail

Cross-sections of the three designs: capillary bundle, annular gap and detail of a matrix of corrugated and flat strip.

OUR LAMINAR FLOW ELEMENTS

The S series covers 0.001 to 990 L/min with six sizes: the smallest, S010, with borosilicate glass multi-capillaries up to 10 L/min, the others with a stainless steel matrix. The R series with a stainless steel matrix reaches 63,700 L/min.

3.2 Hagen-Poiseuille in the LFE

In the classical view the LFE follows the law of Hagen-Poiseuille (2.5). Accordingly, the pressure drop in laminar flow through straight channels is proportional to the volumetric flow and to the dynamic viscosity.

$$\Delta p = C_1 \cdot \eta \cdot \dot{V} \tag{3.1}$$

The constant C_1 contains the geometry alone. For N parallel channels of any cross-section:

$$C_1 = \frac{\mathrm{Po} \cdot L}{2 \cdot N \cdot A \cdot d_{hyd}^2} \quad \text{with} \quad d_{hyd} = \frac{4 \cdot A}{U} \tag{3.2}$$

SYMBOL	MEANING	UNIT
C_1	instrument constant of the friction part	m^{-3}
N	number of parallel channels	–
A	cross-section of one channel	m^2
U	wetted perimeter of one channel	m
d_{hyd}	hydraulic diameter of one channel	m
Po	Poiseuille number of the channel shape	–

CHANNEL CROSS-SECTION	PO	D_{HYD}
circle, diameter d (capillary)	64	d
narrow gap of width s (annular gap)	96	$2s$
equilateral triangle, side a (matrix)	53.3	$a/\sqrt{3}$

For the circle, Po = 64 gives (2.5) again: $C_1 = 8L/(N\pi R^4)$. The Poiseuille number is the product of pipe friction factor and Reynolds number, Po = $\lambda \cdot$ Re. The pipe friction factor describes the pressure loss of a pipe section, $\Delta p = \lambda \cdot (L/d) \cdot \frac{\rho}{2} \bar{c}^2$; for the circular pipe it is $\lambda = 64/\mathrm{Re}$. The channels of a matrix are only approximately equilateral.

Written differently, the quotient is a constant, the ideal relationship of the LFE.

$$\frac{\Delta p}{\dot{V} \cdot \eta} = C_1 = \text{const.} \tag{3.3}$$

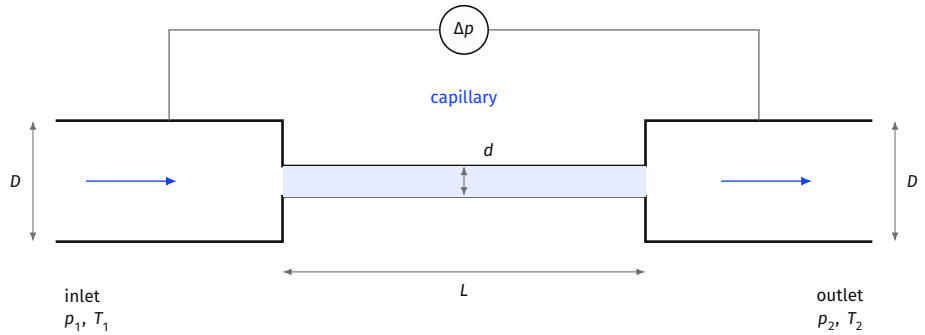
3.3 Pressure tapping on a real LFE

The ideal relationship holds only if the pressure is tapped inside the capillary itself, behind the inlet length – for liquids as for gases. The inlet length grows with the Reynolds number.

$$L_E \approx 0.06 \cdot \mathrm{Re} \cdot d \tag{3.4}$$

SYMBOL	MEANING	UNIT
L_E	inlet length of laminar flow	m
d	diameter of the channel	m

For Reynolds numbers of 30 to 1000 this amounts to 2 to 60 channel diameters. A pressure tap inside the capillary is difficult to realise. Commercial LFEs therefore tap the pressure upstream and downstream of the capillary or matrix, with two bores in the inlet and outlet chambers.



Pressure tapping on a real LFE: upstream and downstream of the capillary. The measured differential pressure includes inlet and outlet.

3.4 Hagenbach correction

With short capillaries, in the limit as thin as an orifice plate, the law of Hagen-Poiseuille alone can no longer hold. On a real LFE the differential pressure is made up of the linear friction part and a quadratic part like the one an orifice produces: the gas is accelerated as it enters the channels. The Hagenbach correction describes the relationship.

$$\Delta p = C_1 \cdot \eta \cdot \dot{V} + C_2 \cdot \rho \cdot \dot{V}^2 \tag{3.5}$$

SYMBOL	MEANING	UNIT
C_2	instrument constant of the acceleration part	m^{-4}
ρ	density of the flowing medium	$\mathrm{kg\,m}^{-3}$

For liquids the equation already describes the influence of density well.

3.5 Compressibility of the gas

With gases, compressibility is added. With the pressure drop in the capillary, the absolute pressure falls and with it the density; the gas expands, and the volumetric flow grows towards the outlet. For isothermal laminar flow this can be calculated in closed form: the friction part is determined by the volumetric flow at the mean pressure.

$$\bar{p} = \frac{p_1 + p_2}{2} \qquad \dot{V}_m = \dot{V}_1 \cdot \frac{p_1}{\bar{p}} = \dot{V}_1 \cdot \frac{2\,p_1}{p_1 + p_2}$$

Referred to the inlet state, (3.5) thus becomes:

$$\Delta p = C_1 \cdot \eta \cdot \dot{V}_1 \cdot \frac{2\,p_1}{p_1 + p_2} + C_2 \cdot \rho_1 \cdot \dot{V}_1^2 \tag{3.6}$$

SYMBOL	MEANING	UNIT
p_1, p_2	absolute pressure upstream and downstream of the LFE	Pa
$\bar{p}$	mean pressure in the LFE	Pa
$\dot{V}_1$	volumetric flow in the inlet state	$m^3\,s^{-1}$
$\dot{V}_m$	volumetric flow at the mean pressure	$m^3\,s^{-1}$
ρ_1	density in the inlet state	$kg\,m^{-3}$

The equation assumes isothermal flow. In the narrow channels of an LFE this is a good approximation.

At a given differential pressure the volumetric flow is therefore only $(p_1 + p_2)/(2\,p_1) = 1 - \Delta p/(2\,p_1)$ times the value without compressibility. The characteristic $\dot{V}_1(\Delta p)$ bends downwards as the differential pressure grows, and the linear part becomes smaller. At 10 mbar differential pressure and 1 bar inlet pressure this amounts to 0.5 %.

3.6 The practical form

Equation (3.6) holds for purely laminar, steady flow without disturbances at inlet and outlet; on a real LFE this is only approximately the case. In practice the Hagenbach form is therefore divided by $\dot{V} \cdot \eta$ and extended by a third-order term, for gases as for liquids.

$$\frac{\Delta p}{\dot{V} \cdot \eta} = C_1 + C_2 \cdot \frac{\dot{V} \cdot \rho}{\eta} + C_3 \cdot \left(\frac{\dot{V} \cdot \rho}{\eta}\right)^2 \tag{3.7}$$

SYMBOL	MEANING	UNIT
C_3	third-order instrument constant	m^{-5}

Compared with (3.3), the density-dependent acceleration part in inlet and outlet now appears in the equation, and the influence of density on real LFEs can be read from it. The group $\dot{V} \cdot \rho / \eta$ is called the *reduced Reynolds number*: for N channels of cross-section A

$$\mathrm{Re} = \frac{\rho \cdot \bar{c} \cdot d_{hyd}}{\eta} = \frac{\dot{V} \cdot \rho}{\eta} \cdot \frac{d_{hyd}}{N \cdot A} \tag{3.8}$$

SYMBOL	MEANING	UNIT
$\bar{c}$	mean velocity in the channel, $\bar{c} = \dot{V}/(N \cdot A)$	$\mathrm{m\,s^{-1}}$

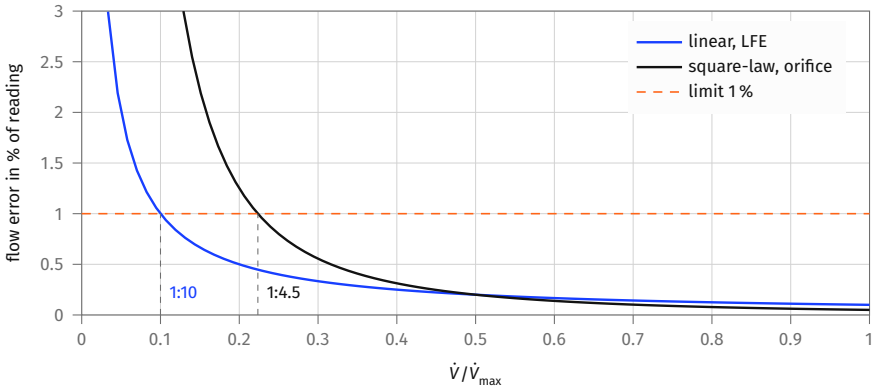
For a fixed geometry, $\dot{V} \cdot \rho / \eta$ is proportional to the Reynolds number. Equation (3.7) thus describes how the resistance of the LFE grows with the Reynolds number; the same idea as $c_w = 24/\mathrm{Re}$ for the sphere. The universal characteristic in section 4.4 rests on it.

3.7 Linear characteristic and turndown

Why the linear characteristic enlarges the turndown is shown by an error calculation. A differential pressure sensor has a fixed error e referred to its full scale. With a linear differential pressure device it passes undiminished into the flow, with a square-law device by half, but with the square of the distance from full scale.

$$\text{linear: } \frac{\delta\dot{V}}{\dot{V}} = \frac{e}{x} \qquad \text{square-law: } \frac{\delta\dot{V}}{\dot{V}} = \frac{e}{2\,x^2} \qquad x = \frac{\dot{V}}{\dot{V}_{\max}} \tag{3.9}$$

If the flow error may be at most ε, the turndown reaches $x_{\min} = e/\varepsilon$ for the linear device and $x_{\min} = \sqrt{e/(2\,\varepsilon)}$ for the square-law device.



Flow error versus flow for a sensor error of 0.1 % of full scale. With a limit of 1 % of reading, the LFE achieves a turndown of 1:10, the orifice 1:4.5.

THE TURNDOWN OF OUR LFES

The S series reaches a turndown of up to 1:50 with the glass capillaries of the S010 and up to 1:20 with a stainless steel matrix. The R series works with 1:10, in special cases up to 1:30.

4 Calibration

Calibration is the starting point of every accurate measurement with an LFE. As chapter 3 has shown, however, the evaluation model also decides under which conditions the LFE can be used later.

Calibration takes place under calibration conditions with a specific calibration medium. Volumetric flow, differential pressure across the LFE, absolute pressure, temperature and, where applicable, humidity are recorded. Density and viscosity are calculated from the measuring conditions. The data points obtained in this way are evaluated with a method that matches the physical model of the later application; this yields the calibration constants of the respective model.

4.1 The Hagen-Poiseuille characteristic

The classical method evaluates according to Hagen-Poiseuille. Velocity and density make the real curve between differential pressure and volumetric flow slightly non-linear. To describe this linearity deviation, a polynomial in the differential pressure is set up and multiplied by the ratio of calibration and operating viscosity.

$$\dot{V} = \left(a_1 \cdot \Delta p + a_2 \cdot \Delta p^2 + \dots\right) \cdot \frac{\eta_{kal}}{\eta} \tag{4.1}$$

SYMBOL	MEANING	UNIT
a_1, a_2	calibration coefficients	$m^3\,s^{-1}\,Pa^{-1}$, …
η_{kal}	viscosity under calibration conditions	Pa s
η	viscosity in operation	Pa s

The characteristic passes through the origin; a constant term is omitted.

Equation (3.6) explains the coefficients. Solved for the volumetric flow and expanded in powers of Δp, it gives under calibration conditions:

$$a_1 = \frac{1}{C_1 \cdot \eta_{kal}} \qquad a_2 \approx -a_1 \cdot \left(\frac{1}{2\,p_{kal}} + C_2 \cdot \rho_{kal} \cdot a_1^2\right)$$

SYMBOL	MEANING	UNIT
p_{kal}	absolute pressure at the LFE inlet during calibration	Pa
ρ_{kal}	density under calibration conditions	$kg\,m^{-3}$

The linear coefficient contains only the viscosity; for it the correction with η_{kal}/η is exact. The quadratic one is negative – the characteristic curves downwards – and has two causes: the compressibility of the gas (first term) and the acceleration at inlet and outlet (second term). Both depend on the pressure and density of the calibration gas. If pressure or gas type change, a_2 no longer applies; sections 4.2 and 4.3 deal with this.

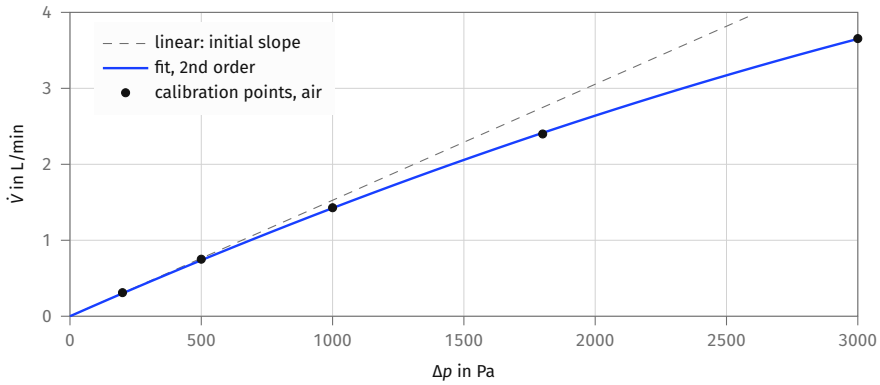
WORKED EXAMPLE: TEMPERATURE CORRECTION

An LFE was calibrated with air at ϑ_{kal} = 21.1 °C. At a volumetric flow of 22.36 L/min, a differential pressure of 10 mbar was measured across it; in the linear range this gives a_1 = 2.236 L/(min · mbar). The LFE is used in air at 25 °C. The viscosities follow from the Sutherland equation (7.3). At 10 mbar, (4.1) without the quadratic term gives:

$$\dot{V} = 2.236\ \mathrm{L/(min \cdot mbar)} \cdot 10\ \mathrm{mbar} \cdot \frac{\eta(294.25\ \mathrm{K})}{\eta(298.15\ \mathrm{K})}$$

$$= 22.36\ \mathrm{L/min} \cdot \frac{18.187\ \mu\mathrm{Pa\,s}}{18.372\ \mu\mathrm{Pa\,s}} = 22.13\ \mathrm{L/min}$$

Without correction, the measurement would read about 1 % high.

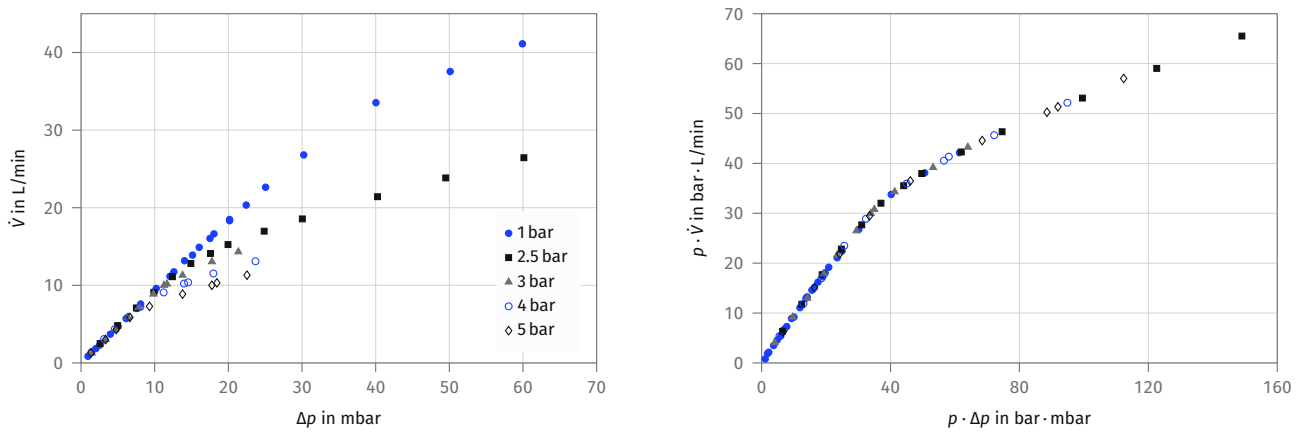


Hagen-Poiseuille characteristic of an LFE calibrated with air, and second-order fit through the origin to illustrate the curvature. The dashed line shows how far the curve departs from the initial slope.

This evaluation can be transferred to other operating states only to a limited extent. An atmospheric calibration with air leads to large measurement deviations at higher pressures or with a gas of very different density and viscosity. As a rule it is transferable only as long as the largest Reynolds number of the calibration is not exceeded in operation.

4.2 One LFE at different operating pressures

The following figure shows measured values of the same LFE with air at 1, 2.5, 3, 4 and 5 bar absolute. At small differential pressure all series lie on the same straight line: the viscosity hardly depends on pressure, and Hagen-Poiseuille applies. As the differential pressure grows, the characteristic bends, and the earlier the higher the pressure; the density-dependent part acts more strongly.



Left: volumetric flow versus differential pressure at five absolute pressures. Right: the same measuring points, volumetric flow and differential pressure each multiplied by the absolute pressure. All series fall onto one curve.

At 10 mbar the LFE delivers about 19 % less volumetric flow at 5 bar than at 1 bar. An atmospheric calibration, transferred according to Hagen-Poiseuille, would read about 24 % high there.

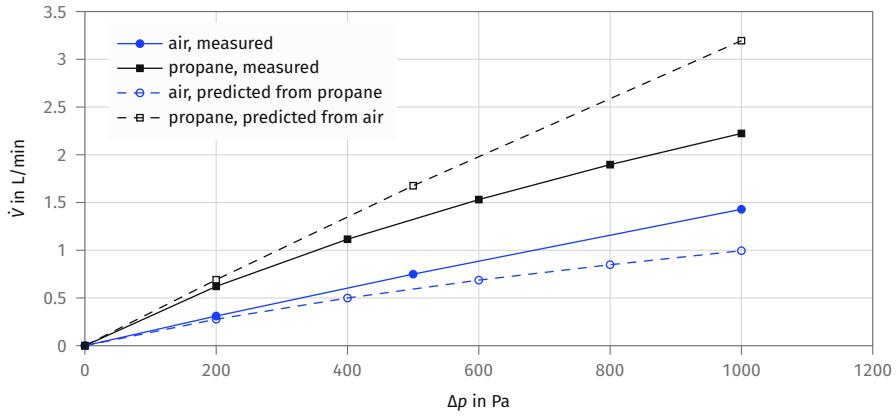
The right-hand plot explains the finding. The Reynolds number (3.8) grows with the density, and by the gas equation (7.1) density is proportional to pressure and inversely proportional to temperature.

$$\mathrm{Re} \sim \frac{\dot{V} \cdot \rho}{\eta} \sim \frac{p \cdot \dot{V}}{\eta \cdot T} \tag{4.2}$$

The same Reynolds number means the same flow pattern. To stay at 5 bar within the range in which the LFE works laminar at 1 bar, only one fifth of the volumetric flow may pass, and only one fifth of the differential pressure span is used. In general: reduce the differential pressure span by the factor by which the absolute pressure exceeds atmospheric pressure.

4.3 Transfer to another gas

Calibration data of an LFE always refer to the calibration conditions; gas type and operating pressure enter the calibration constants. Air under atmospheric conditions is the usual calibration medium for reasons of cost. The following figure shows what happens when such a calibration is transferred to propane via the viscosity ratio (4.1) alone, and vice versa.



The same LFE with air and with propane. Filled markers: measured. Open markers: predicted according to Hagen-Poiseuille from the calibration with the other gas.

At 1000 Pa the air calibration predicts 3.20 L/min for propane; 2.22 L/min are measured. The prediction is 44 % too high. Conversely, the propane calibration underestimates the air flow by 30 %. It follows:

- An atmospheric calibration, transferred to overpressure according to Hagen-Poiseuille, is the further off the higher the operating and differential pressures are (section 4.2).
- Between gas types with strongly different properties, the transfer misses the flow by tens of percent at the same operating and differential pressure, between air and propane by up to 44 %.
- A calibration can be transferred to another gas only if the largest Reynolds number of the calibration roughly matches that in operation and density and viscosity differ by no more than 10 %.
- For precision applications the calibration gas should match the operating gas. If air is measured at a higher pressure, calibration should take place at that pressure, within ±10 % of the operating pressure.

These errors are caused by compressibility and acceleration effects that the Hagen-Poiseuille approach does not take into account; they make the result depend on density. A model based on the Reynolds number describes it better: the universal characteristic.

4.4 The universal characteristic

In a comprehensive study in 1980 and 1981, 160 LFE calibration data sets were evaluated in the USA. It showed that an equivalent description can be derived from relationships (3.5) to (3.7) which represents all measurements on a single characteristic.

The step there is short. Multiplying the Hagenbach form (3.5) by ρ/η^2 leaves only two groups.

$$\underbrace{\frac{\Delta p \cdot \rho}{\eta^2}}_{X} = C_1 \cdot \underbrace{\frac{\dot{V} \cdot \rho}{\eta}}_{Y} + C_2 \cdot \left(\frac{\dot{V} \cdot \rho}{\eta}\right)^2 \tag{4.3}$$

Density and viscosity no longer appear separately in the equation: whoever forms X and Y obtains the same curve for every gas and every pressure. Y is the reduced Reynolds number from (3.8), X the matching pressure quantity. Both also follow from a dimensional analysis when the geometry of the LFE is fixed. The characteristic is set up as a polynomial.

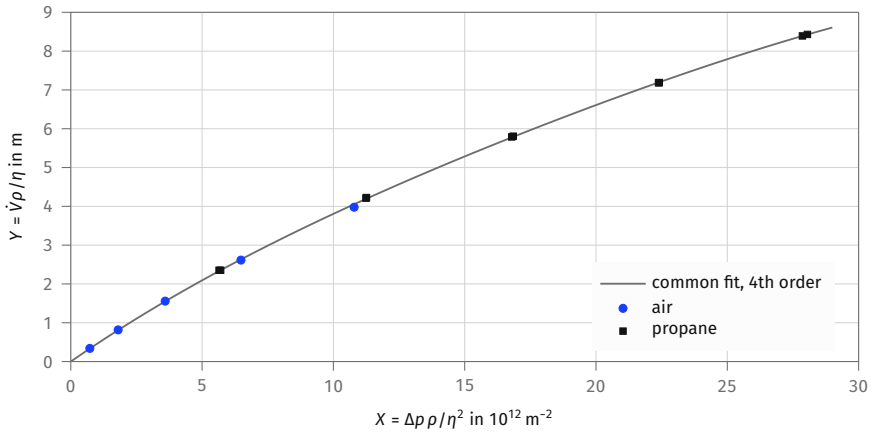
$$Y = b_1 \cdot X + b_2 \cdot X^2 + b_3 \cdot X^3 + \ldots \tag{4.4}$$

With the gas properties and flow data as variables:

$$\frac{\dot{V} \cdot \rho}{\eta} = b_1 \cdot \frac{\Delta p \cdot \rho}{\eta^2} + b_2 \cdot \left(\frac{\Delta p \cdot \rho}{\eta^2}\right)^2 + b_3 \cdot \left(\frac{\Delta p \cdot \rho}{\eta^2}\right)^3 + \ldots \tag{4.5}$$

SYMBOL	MEANING	UNIT
X	pressure quantity of the universal characteristic	m^{-2}
Y	reduced Reynolds number	m
b_1, b_2, b_3	coefficients of the universal characteristic	m^3, m^5, m^7

In the linear limit $b_1 = 1/C_1$: the first order is the Hagen-Poiseuille constant. A constant term is omitted because the characteristic passes through the origin.

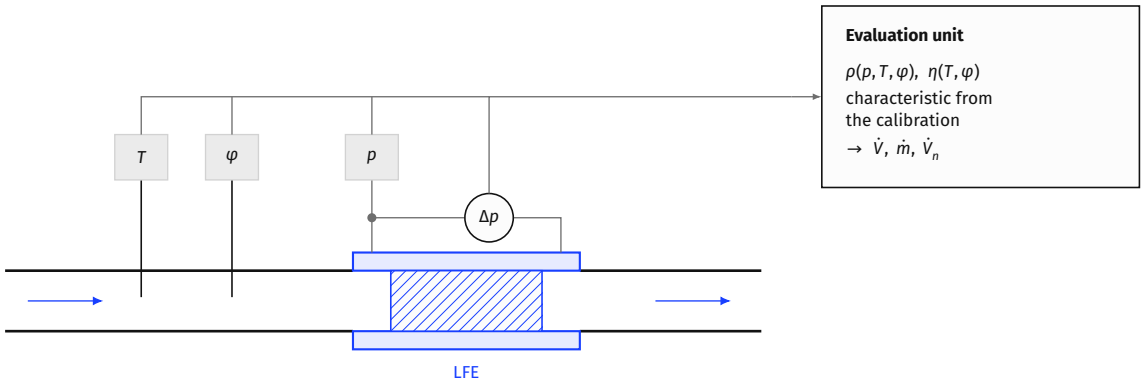


The measurements from the air and propane comparison in the universal plot. Both gases lie on one curve; shown is a common fourth-order polynomial through both series.

Like the Hagen-Poiseuille approach, this polynomial describes the deviation from the ideal state in the first order with the higher orders, this time taking density into account. The authors around David Todd of the Naval Primary Standards Laboratory in San Diego, California, and we ourselves have tested this evaluation with various gases at different pressures. It gives considerably better results and predictions than the Hagen-Poiseuille evaluation and reduces the deviations on changes of operating pressure and gas type to below 1 %.

In return it needs a considerably higher polynomial order to describe the characteristic with mathematical accuracy. An essential precondition remains that the largest Reynolds number of the calibration is not exceeded in operation. The characteristic is clearly curved; outside the calibrated range, extrapolation quickly brings high uncertainties here too.

5 Evaluation in operation



Evaluation scheme. Differential pressure, absolute pressure, temperature and, where applicable, humidity are recorded at the LFE; the evaluation unit calculates gas properties and flow from them.

Evaluation in operation is the reverse of calibration. Differential pressure, absolute pressure, temperature and, where applicable, humidity are recorded. From these measured quantities the evaluation calculates density and viscosity and inserts them, together with the differential pressure, into the model used for calibration. The result is the volumetric flow. Both methods proceed in the same way.

5.1 Hagen-Poiseuille model

$$\dot{V} = \left(a_1 \cdot \Delta p + a_2 \cdot \Delta p^2\right) \cdot \frac{\eta_{kal}}{\eta} \tag{5.1}$$

5.2 Universal characteristic

$$\dot{V} = \frac{\eta}{\rho} \cdot \left(b_1 \cdot X + b_2 \cdot X^2 + b_3 \cdot X^3\right) \qquad \text{with} \qquad X = \frac{\Delta p \cdot \rho}{\eta^2} \tag{5.2}$$

The coefficients are given in the calibration record; they are determined by the method of least squares or an equivalent procedure. The deviation between polynomial and measured flow should as a rule be below 0.1 % in the calibration; if it is higher, a higher order is to be used. The conversion to mass or standard volumetric flow follows the continuity equation (chapter 6).

UNIFLOW IN OUR EVALUATION UNITS

Our evaluation units CoM4.Flow-Eco, -Smart and -Pro and the CoM4.Flow-Quattro calculate for an LFE according to Hagen-Poiseuille or the UniFlow method adapted by TetraTec, an evaluation via the universal characteristic (section 4.4). It includes the density in the linearisation and is used above all at higher pressures.

6 Flow quantities and their conversion

Gases are compressible, so gas flows depend on density. The continuity equation links the usual quantities; the volumetric flows can be converted into each other at any time via the density ratio.

$$\rho \cdot \dot{V} = \dot{m} = \rho_n \cdot \dot{V}_n \tag{6.1}$$

QUANTITY	MEANING	SI UNIT
Actual volumetric flow $\dot{V}$	cross-section times mean flow velocity, volume per time in the state at the measuring point	m^3/s
Mass flow $\dot{m}$	actual volumetric flow times density in the operating state	kg/s
Standard volumetric flow $\dot{V}_n$	mass flow divided by the density at defined standard conditions. Because these are fixed, it stands in a fixed ratio to the mass flow.	m^3/s
Reference volumetric flow	like the standard volumetric flow, but referred to freely chosen reference conditions, such as those of the test piece	m^3/s

Pressure and temperature determine the density ratio, for a real gas also the compressibility factor (7.1).

$$\dot{V}_n = \dot{V} \cdot \frac{p}{p_n} \cdot \frac{T_n}{T} \cdot \frac{Z_n}{Z} \tag{6.2}$$

SYMBOL	MEANING	UNIT
p, T	absolute pressure and temperature in the operating state	Pa, K
p_n, T_n	standard pressure and standard temperature	Pa, K
Z, Z_n	compressibility factor; ideal gas $Z = 1$	–

For dry gas. For humid air, (6.1) applies with the density from (7.2); the standard volumetric flow then refers to the dry gas.

WORKED EXAMPLE: STANDARD AND MASS FLOW

The LFE from the example in section 4.1 measures 22.13 L/min of air at 25 °C and an absolute pressure of 980 mbar. Referred to the standard conditions 1000 mbar and 20 °C, with $Z = Z_n = 1$ and the density from (7.1):

$$\dot{V}_n = 22.13\ \text{L/min} \cdot \frac{980\ \text{mbar}}{1000\ \text{mbar}} \cdot \frac{293.15\ \text{K}}{298.15\ \text{K}} = 21.32\ \text{L/min}$$

$$\rho_n = \frac{100\,000\ \text{Pa}}{287.05\ \text{J/(kg·K)} \cdot 293.15\ \text{K}} = 1.1884\ \text{kg/m}^3$$

$$\dot{m} = \rho_n \cdot \dot{V}_n = 25.34\ \text{g/min}$$

WHICH STANDARD CONDITION APPLIES

Our data sheets state the standard condition as a separate item, usually 1000 mbar, 20 °C and 0 % r. h. The standard state according to DIN 1343 is 1013.25 mbar and 0 °C. The two differ by almost 9 %. Which condition applies is stated in the data sheet of the respective instrument.

7 Gas properties and sensors

Gas properties and sensors decide on the measuring accuracy and on whether a calibration can be transferred to other operating states or gas types.

7.1 Density

Density is calculated from temperature, absolute pressure and, where applicable, humidity. Many gases deviate from ideal behaviour. For precise evaluations, above all at higher pressures, the compressibility factor therefore belongs in the calculation.

$$\rho = \frac{p}{Z \cdot R_i \cdot T} \tag{7.1}$$

SYMBOL	MEANING	UNIT
R_i	specific gas constant; dry air 287.05	J kg^{-1} K^{-1}
Z	compressibility factor; ideal gas Z = 1	–

For humid air under atmospheric conditions the ideal gas equation with the water vapour fraction is a good approximation, with R_i the gas constant of dry air.

$$\rho = \frac{p}{R_i \cdot T} \cdot \left(1 - 0.378 \cdot x_v\right) \qquad \text{with} \qquad x_v = \frac{\varphi \cdot p_s(T)}{p} \tag{7.2}$$

SYMBOL	MEANING	UNIT
x_v	mole fraction of water vapour	–
φ	relative humidity	–
$p_s(T)$	saturation vapour pressure of water; about 2340 at 20 °C	Pa

0.378 = 1 – M_W/M_L with the molar masses of water, M_W = 18.02 g/mol, and dry air, M_L = 28.96 g/mol: water vapour is lighter than air, so humid air is lighter than dry air. The CIPM formula for the density of moist air (CIPM-2007), developed with the participation of the PTB, calculates it exactly.

The equation shows how strongly a sensor error affects the density.

$$\frac{\delta\rho}{\rho} = \frac{\delta p}{p} - \frac{\delta T}{T} - 0.378 \cdot \frac{p_s}{p} \cdot \delta\varphi$$

MEASURED QUANTITY	SENSOR ERROR	ERROR IN DENSITY
Temperature	1 K	0.34 %
Absolute pressure	3 mbar	0.30 %
Relative humidity	45 % r. h.	0.39 %

Calculated for air at 20 °C and 1013.25 mbar.

As a rule of thumb, a temperature error of 1 K, a pressure error of 3 mbar and a humidity error of 45 % weigh about the same, each with about 0.3 to 0.4 % in density. This shows the weighting of the sensors: for air, the humidity measurement carries the least weight.

7.2 Viscosity

Like density, viscosity is calculated from the measured quantities, from temperature and, for air, also from humidity. Dry approaches without pressure influence, such as Sutherland or Daubert and Danner, work very well for pure gases up to about 7 bar. For air, a model with humidity improves accuracy, for example after Kestin and Whitelaw.

$$\eta(T) = \frac{\beta \cdot T^{3/2}}{T + S} \tag{7.3}$$

SYMBOL	MEANING	UNIT
β	constant; air $1.458 \cdot 10^{-6}$	$\mathrm{kg\,m^{-1}\,s^{-1}\,K^{-1/2}}$
S	Sutherland temperature; air 110.4	K
T	absolute temperature	K

The values for air are those of the 1976 US Standard Atmosphere and of ISO 2533. At 20 °C they give η = 18.1 µPa s. Older documents calculate in micropoise; their numerical values are ten times larger, since 1 µPa s = 10 µP.

Differentiated with respect to temperature:

$$\frac{\delta\eta}{\eta} = \left(\frac{3}{2\,T} - \frac{1}{T + S}\right) \cdot \delta T$$

At 20 °C that is 0.26 % per kelvin. A humidity error of 45 % r. h. has an effect of about 0.2 %.

If the temperature is not taken into account, the viscosity error enters the volumetric flow to the same extent via (4.1). In the range of about 10 to 30 °C:

TEMPERATURE DEVIATION δT	1 K	5 K	10 K
Deviation in volumetric flow	0.26 % rdg.	1.3 % rdg.	2.6 % rdg.

TEMPERATURE AND HUMIDITY IN THE GAS STREAM

For the gas properties, our temperature and humidity sensor HUMTMP records both quantities in the gas stream; for temperature alone we offer the Pt100 probe WIT-S.

8 Application: testing cross-sections

A widespread test method for gaps, annular gaps, nozzles, openings and orifices is to pass air through them and measure the volumetric or mass flow. Particularly good results are obtained when the test piece is operated like a critical nozzle. Sonic velocity then sets in at the narrowest cross-section as soon as the ratio of outlet to inlet pressure falls below the critical value.

$$\frac{p_2}{p_1} \le \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa}{\kappa - 1}} = 0.528 \qquad \text{(air, } \kappa = 1.4\text{)} \tag{8.1}$$

The inlet pressure must therefore be at least 1.89 times the outlet pressure; a pressure ratio above 2:1 keeps sufficient margin. In critical operation the volumetric flow at the inlet of the test piece is almost independent of inlet and outlet pressure. It depends essentially on the cross-sectional area and the speed of sound.

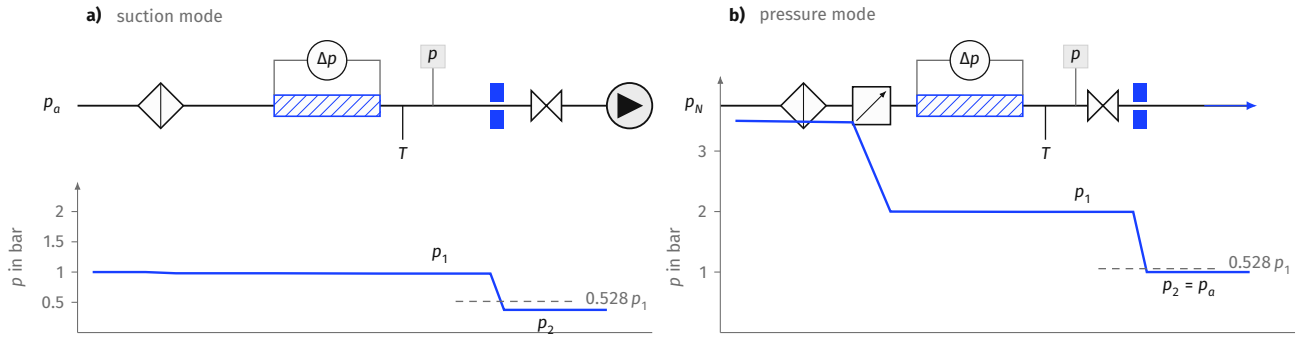
$$\dot{V}_1 = C_D \cdot A \cdot a_1 \cdot \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa+1}{2(\kappa-1)}} \quad \text{with} \quad a_1 = \sqrt{\kappa \cdot R_i \cdot T_1} \tag{8.2}$$

$$\dot{V}_1 = 0.579 \cdot C_D \cdot A \cdot a_1 \quad \text{(air)}$$

SYMBOL	MEANING	UNIT
p_1, p_2	absolute pressure upstream and downstream of the test piece	Pa
κ	isentropic exponent; air 1.4	–
$\dot{V}_1$	volumetric flow at the inlet of the test piece	$m^3\,s^{-1}$
C_D	discharge coefficient of the test piece	–
A	narrowest cross-section	m^2
a_1	speed of sound at the inlet; air at 20 °C 343	$m\,s^{-1}$
T_1	temperature at the inlet	K

Pressure does not appear in the equation, and the volumetric flow is proportional to the area. Only the pressure ratio matters for the effect: it makes no difference whether the test is carried out with vacuum at the outlet or with overpressure at the inlet of the test piece.

Between measuring point and test piece, for example in the test piece adapter, pressure drops can make the conditions at the test piece differ from those at the flow meter. For correction, a reference sensor measures the inlet pressure of the test piece. The following figure shows the pressure curve in both arrangements.



Pressure curve along a test line. a) Atmospheric suction with a vacuum pump. b) Pressure mode with pressure regulator and free discharge to atmosphere. In both: filter, LFE with a differential pressure below 5 mbar, reference pressure p and temperature T, shut-off valve and the test piece (blue). p_a atmospheric pressure, p_N supply pressure.

With a precise volumetric flow meter, the smallest differences in cross-sectional area can thus be detected quickly. In the pairing of miniature pistons and cylinders the method has been working for years with cycle times of a few seconds.

For the highest accuracy requirements, the measurement needs corrections for temperature, pressure and flow. The physical behaviour of the test piece, such as thermal expansion, and of the test medium, the temperature and pressure dependence of viscosity, density and speed of sound, changes the flow even with the same cross-sectional area. Which dependences act depends on the physics of the test piece type.

The method meets the five requirements for a production test method. It is

- **reproducible**,
- **efficient**,
- **robust**,
- **free of reaction on the test piece**
- and **suited to computer evaluation**.

> **IN PRESSURE MODE THE LFE WORKS UNDER PRESSURE TOO**
>
> In arrangement b the LFE sits at about 2 bar. What chapter 4 shows applies here: calibration takes place at this pressure, or the evaluation includes the density.

9 Summary

Laminar flow elements offer all the advantages of fast differential pressure devices without their drawback, the small turndown of the square-root characteristic. They can be evaluated with very good repeatability even over large turndowns. Over a turndown of 1:10, differential pressure sensors with the usual accuracy of 0.1 % of full scale are sufficient to determine the gas flow to 1 % of reading (section 3.7).

The dependence on viscosity calls for an evaluation that includes density and viscosity, and for a calibration under conditions that match operation. Where operating pressure or gas type differ, the universal characteristic carries the evaluation, as long as the largest Reynolds number of the calibration is not exceeded.

We supply complete flow measuring systems based on LFEs. With the evaluation units CoM4.Flow-Eco, -Smart and -Pro, the measurement uncertainty (k = 2) over a turndown of 1:10 is ±1.5, ±1 or ±0.65 % of reading, depending on the instrument. The figures apply to the chain of evaluation unit and LFE from TetraTec, and to air.

Measuring instruments working on this principle have been used since 1987 in the nozzle production of an industrial customer for the geometric measurement of nozzles and for production monitoring.

Where this physics is applied in our work

Calibration laboratory

DAkkS and factory calibrations for quantities of gas flow measurement, in the laboratory or on site. Accredited to EN ISO/IEC 17025 since 1999, registration no. D-K-17589-01-00.

The accreditation certificate lists three gas ranges: N_2, H_2, CH_4, He and Ar from 10 L/h to 9 m^3/h, CO from 10 L/h to 1.5 m^3/h, propane from 0.5 L/h to 600 L/h. On-site calibration forms a section of its own in the certificate.

Measurement and test technology

Sensors and systems for air and gas flow, pressure, temperature, humidity and leak tightness – laminar flow elements, differential pressure devices to EN ISO 5167, capillary calibrated leaks and complete test rigs.

Engineering

Tailored test and measuring systems, developed together with you and integrated into your production.

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About this document

Issue	Version 1.0, 2026-11-12
Basis	Technical paper “Gas-Durchflussmessung mit Laminar Flow Elementen” by Dipl.-Phys. Karl Ilg, TetraTec Instruments, 2014; revised and extended. It incorporates the sheet “Durchflussmessung mit LFEs” of 13 February 2018, which this paper replaces.
Text and figures	Own work of TetraTec Instruments GmbH

The measured values in the diagrams of chapter 4 are taken from that paper.

This paper replaces neither a textbook nor a sizing calculation. It reflects our present knowledge and serves as orientation; no liability for completeness or accuracy is accepted. What governs the layout of a measuring set-up are the applicable standards and the data of the instrument used. For a particular case, please talk to us.